Paper: Introductory Mathematical Methods for Economics

Course: B. A.(Hons.) Economics I Year

Year: 2022

ECON002: Introductory Mathematical Methods for Economics

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Duration : 3 Hours **Maximum Marks : 90**

SEMESTER–1

2022

There are six questions in all. All questions are compulsory.

Q. 1. Answer any two of the following : 2×4=8

(a) Find the *solution/solution* set for the following :

(i) $|x^2 + 6x + 16| < 8$

Ans. $|x^2 + 6x + 16| < 8$

$\Rightarrow \quad -8 < x^2 + 6x + 16 < 8$

$\Rightarrow \quad -8 < x^2 + 6x + 16$ and $x^2 + 6x + 16 < 8$

$\Rightarrow \quad 0 < x^2 + 6x + 24$ and $x^2 + 6x + 8 < 0$

$\Rightarrow \quad 0 < (x +)^2 + 15$ and $x^2 + 4x + 2x + 8 < 0$

$\Rightarrow \quad 0 < (x + 3)^2 + 15$ and $(x + 4)(x + 2) < 0$

$\Rightarrow \quad x \in R$ and $x \in (-4, -2)$

(ii) $\log_3 (x + 6) + \log_3 (x - 2) = 2$

Ans. $\log_3 (x + 6) + \log_3 (x - 2) = 2$

$\Rightarrow \quad \log_3 ((x + 6)(x - 2)) = 2$

$\Rightarrow \quad (x + 6)(x - 2) = 32$

$\Rightarrow \quad (x + 6)(x - 2) = 9$

$\Rightarrow \quad x^2 + 4x - 21 = 0$

$\Rightarrow \quad x^2 + 7x - 3x - 21 = 0$

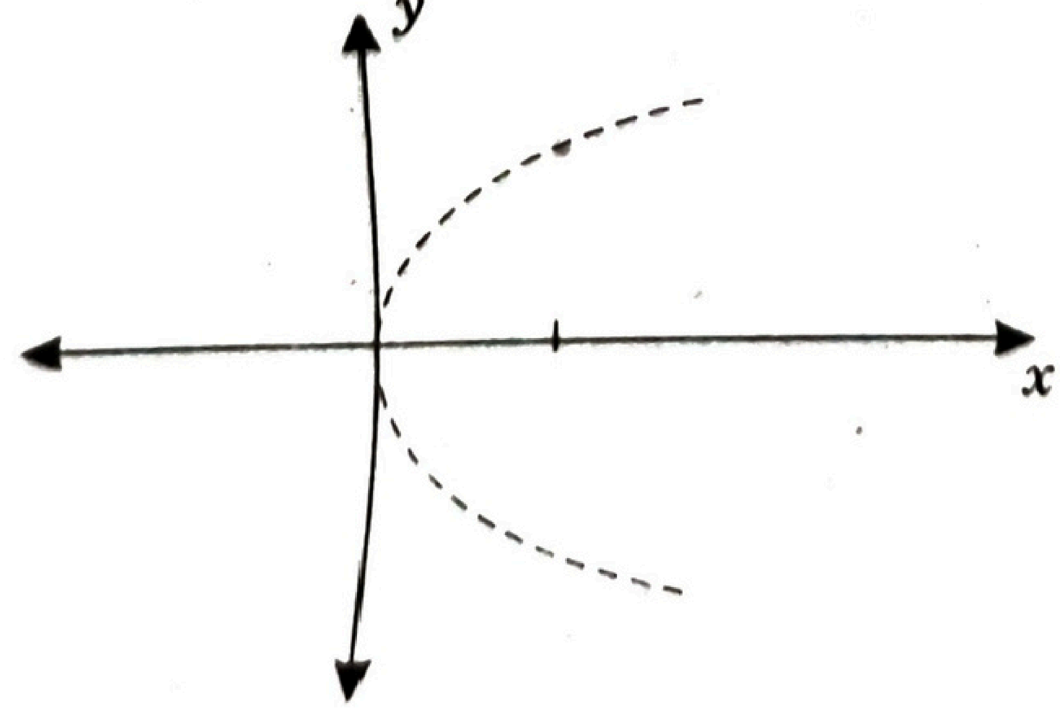
$\Rightarrow \quad (x + 7)(x - 3) = 0$

$\Rightarrow \quad x = -7$ or $x = 3$

(b) Does any of the following drawn on a rectangular coordinat plane represent a function $y = f(x)$? Why or why not?

(i) $y^2 = 3x$

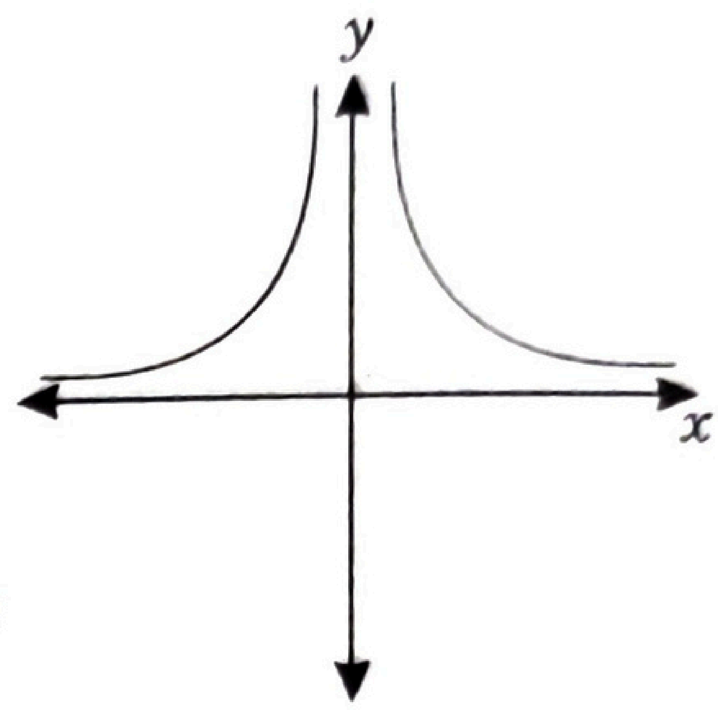
Ans. $y^2 = 3x$



No, since for an $x \in R$ (in the domain), there exist two images of it.

For example for $x = \frac{1}{3}, y = \pm 1$

(ii) $y = \frac{1}{|x|}$



Ans.

$$y = \frac{1}{|x|}$$

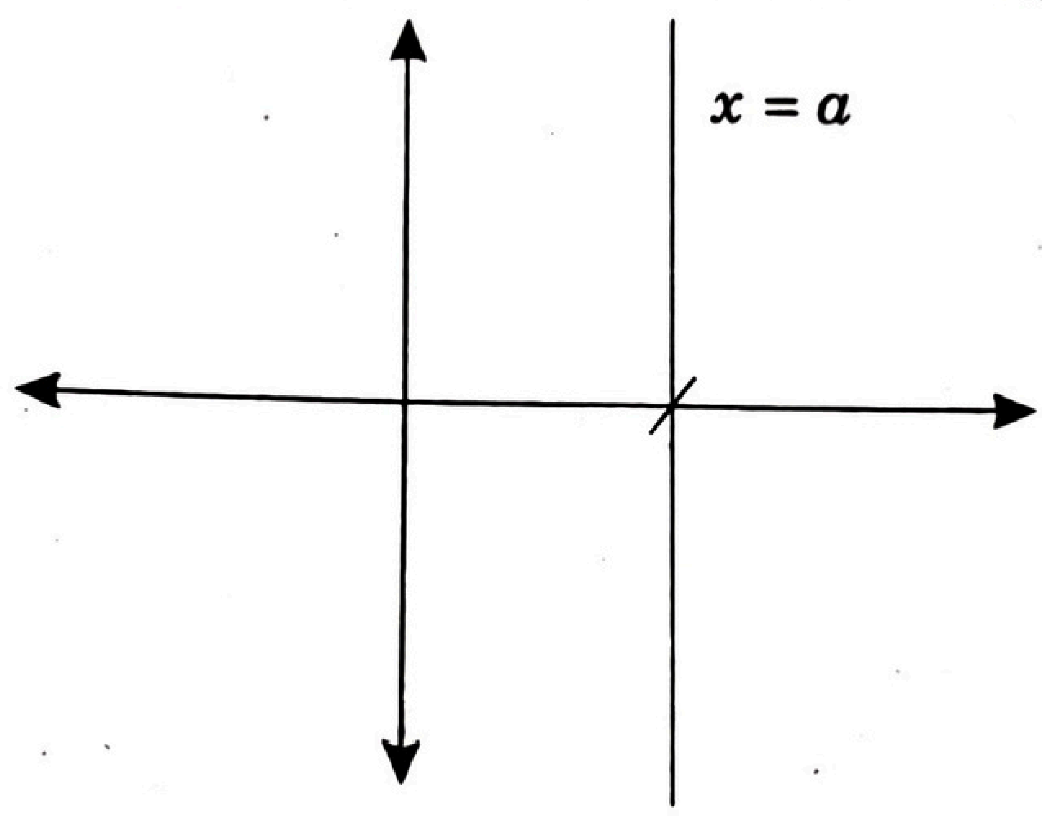
for $x < 0,\ yx = 0$

for $x < 0,\ -xy = 1$

Yes, since it satisfies the condition that for an $x \in R$, there exists a unique image of it in the co-domain.

(iii) A vertical straight line Substantiate your answer with the help of a graph in each case.

Ans. No, since for a single value of x (which is $a \in R$), there exist infinite y.



(c) For each of the following propositions *P* and *Q*, state whether *P* is a necessary condition, or a sufficient condition, or both necessary and sufficient for *Q* to be true?

(i) P : Ail's vehicle has four wheels.

Q : Ali has a car.

Ans. P is a necessary condition.

(ii) P : The series $\sum_{n}^{\infty} a_n$ is convergent

Q : $\lim_{x \to \infty} a_n = 0$

Ans. Sufficient.

(iii) P : $x = (-8)^{1/3}, x \in R$

Q : $x = -2$

Ans. Sufficient.

(iv) P : anumber n is odd

Q : n is a prime number strictly greater than 2. 2×4=8

Ans. Necessary

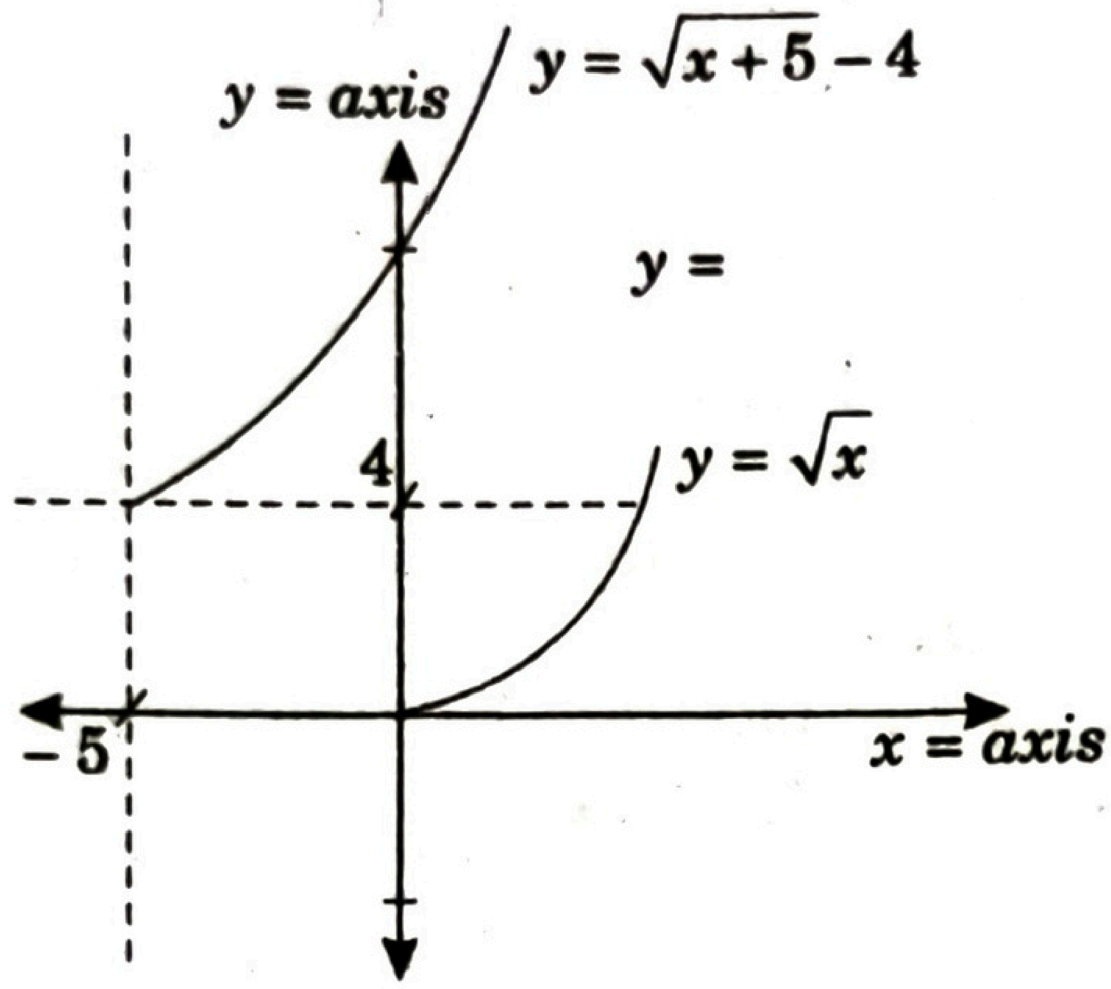
Q. 2. Answer any four of the following :

(a) Draw the graph of $y = \sqrt{x+5} - 4$ using the graph of $y = \sqrt{x}$.

Ans.

$$y = \sqrt{x+5} - 4$$

$$y = \sqrt{x}$$



(b) Is there a solution to the equation $x^2 + 2x - 2 = 0$: is the solution unique?

Ans. $x^2 + 2x - 2 = 0$

Let $f(x) = x^3 + 2x - 2$

$\Rightarrow$ $f'(x) = 3x^3 + 2$

Since $f'(x)$ is always positive for all $x \in R$, f is strictly increasing.

Also for $x = 0,\ f(0) = -2$

So, it attains a negative value, hence it must cut the x-axis. Also since it i[s] monotonically increasing, it will cut the x-axis just once.

Hence $x^2 + 2x - 2 = 0$ has a unique real solution.

(c) A function is given as $f(x) = 5 + 2^{-x^2}$,

(i) Find its domain and range.

Ans. $f(x) = 5 + e^{-x^2}$

Domain $= R$

Since $e^{-x^2} > 0$

$\Rightarrow$ $5 + e^{-x^2} > 5$

Hence range $= (5, \infty)$

(ii) Find its horizontal asymptote(s), if any.

Ans. Horizontal Asymptote

$$f(x) = \lim_{x \to \infty} 5 + e^{-x^2}$$

$$= 5 + \lim_{x \to \infty} \frac{1}{e^{x^2}} = 5$$

$\Rightarrow$ $f(x) = 5$ is the horizontal asymptote.

(d) Find the integer roots of the following equation :

$$3x^4 - 12x^3 - x^2 + 4x = 0$$

Ans. $3x^4 - 12x^3 - x^2 + 4x = 0$

$\Rightarrow$ $3x^3(x-4) - x(x-4) = 0$

$\Rightarrow$ $(3x^3 - x)(x-4) = 0$

$\Rightarrow$ $x(3x^3 - 1)(x-4) = 0$

$\Rightarrow$ $x = 0,\ x = 4,\ x = \frac{1}{\sqrt{3}}$ or $x = -\frac{1}{\sqrt{3}}$

Hence the integer roots are $x = 0$, or $x = 4$.

(e) test the following for convergence :

(i) The sequence $S_n = \frac{n^2 - 1}{n^2 - n}$ 16

Ans. $S_n = \frac{n^2 - 1}{n^2 - n}$

$$\Rightarrow \lim_{n \to \infty} S_n = \lim_{n \to \infty} \frac{1 - \frac{1}{n^2}}{1 - \frac{1}{n}}$$

$$= 1$$

Since it appraches some limit, it is convergent.

(e) The series $\sum_{n=1}^{\infty} (-1)^n (2)^{1/n}$

Ans. $\sum_{n=1}^{\infty} (-1)^n (2)^{1/2}$

Consider $2^{\epsilon} = e \log 2^{\epsilon}$

$\Rightarrow$ $2^{\epsilon} = e^{\epsilon} \log_e 2$

$\Rightarrow$ $2^{\epsilon} \approx 1 + \epsilon \log_e 2$

Now, $2^{\epsilon_0} - 2^{\epsilon_1} = \epsilon_0 \log 2 - \epsilon_1 \log 2$

$$= \left[\frac{1}{(n)} - \frac{1}{(n+1)}\right] \log 2$$

Now, $\frac{1}{n^2 + 1} \approx \frac{1}{n^2}$

Now, $\Sigma \frac{1}{n^2} \approx \frac{\pi^2}{6}$

Hence the sum of the terms (series) is finite. hence the series converges.

Q. 3. Answer any three of the following : **3×5=15**

(a) Find the following limits :

(i) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$

Ans. $\lim\limits_{x \to -\infty} \dfrac{\dfrac{\sqrt{9x^6 - x}}{x^3}}{\dfrac{x^3 + 1}{x^3}}$

$\Rightarrow \quad \lim\limits_{x \to -\infty} \dfrac{\sqrt{\dfrac{9x^6 - x}{x^6}}}{1 + \dfrac{1}{x^3}}$

$\Rightarrow \quad \lim\limits_{x \to -\infty} \dfrac{\sqrt{9 \dfrac{1}{x^5}}}{1 + \dfrac{1}{x^3}}$

$\Rightarrow \quad \sqrt{9}$

$\Rightarrow \quad 3$

(ii) $\lim\limits_{x \to 0} \dfrac{a^{3x} - a^{2x} - a^x + 1}{2x^2}$

Ans. $\lim\limits_{x \to 0} \dfrac{a^{2x}(a^x - 1) - 1(a^x - 1)}{2x^2}$

$\Rightarrow \quad \lim\limits_{x \to 0} \dfrac{(a^{2x} - 1)(a^x - 1)}{2x^2}$

$\Rightarrow \quad \lim\limits_{x \to 0} \dfrac{a^{2x} - 1}{2x} \lim\limits_{x \to 0} \dfrac{a^x - 1}{x}$

$\Rightarrow \quad \log_e a . \log_e a$

(b) The equation of the demand curve is given as :

$$D(P) = \frac{A}{P^B}$$

where, A and B are positive contants and P is the price.

Ans. $$D(P) = \frac{A}{P^B}$$

(i) Find the price elasticity of demand.

Ans. $$D'(P) = \frac{-BA}{P^{B+1}}$$

Elasticity of demand $$= (-1)D'(P)\cdot\frac{P}{D(P)}$$

$$= \frac{BA}{P^{B+1}}\cdot\frac{P\cdot P^B}{A}$$

$$= B$$

(ii) Find elasticity of $T(P)$ with respect to [redacted] where $T(P) = P \,.\, D(P)$.

Ans.

$$T(P) = P\,.\,D(P)$$

$$\frac{dT(P)}{dp} = PD'(P) + D(P)$$

$$= P\left(\frac{-BA}{P^{B+1}}\right) + \frac{A}{P^B}$$

$$= \frac{-BA}{P^B} + \frac{A}{P^B}$$

Elasticity of $T(P)$ with respect to P

$$= \frac{A-AB}{P^B}\cdot\frac{P}{P\cdot\frac{A}{P^B}} = \frac{A-AB}{A} = 1-B$$

(c) The line $2x - y + 1 = 0$ is tangent to a circle at (2, 5). Moreover, the centre of the circle is on the line $x + y = 9$. Find the equation of the circle.

Ans. Slope of the tangent = 2

Let the centre be (a, b)

Slope of the line segment joining $A\,(a, b)$ and $B\,(2, 5)$ would be $\frac{5-b}{2-a}$

Since AB and the line $2x - y + 1 = 0$ are perpendicular to each other,

$$\frac{5-b}{2-a}\times 2 = -1$$

$$\Rightarrow \quad 10 - 2b = -2 + a$$

$$\Rightarrow \quad a + 2b = 12 \qquad \text{...(i)}$$

and since (a, b) lies on the line $x + y = 9$, we have $a + b = 9$ ...(ii)

Solving (i) and (ii)

$$b = 3$$

and $$a = 6$$

So the radius is $\sqrt{(6-2)^2 + (3-5)^2}$

$$= \sqrt{16+4}$$

$$= \sqrt{20}$$

So the equation of the circle is

$$(x-2)^2 + (y-5)^2 = 20$$

$$\Rightarrow \quad x^2 - 4x + y^2 - 10y + 9 = 0$$

(d) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ around $x = 0$ and use it to obtain an estimate of $\sqrt{0.95}$. Also find an upper limit for the error of approximation.

Ans. $f(x) = (1-x)^{1/2}$

Linear approximation entails :

$$f(x) = f(a) + f'(a)(x-a) \text{ where } a = 0 \text{ here.}$$

$$\Rightarrow \quad f(x) \approx 1 + (-1)\left(\frac{1}{2}\right)^{-\frac{1}{2}}(x-0)$$

$$\Rightarrow \quad f(x) \approx 1 - \frac{1}{2}x$$

$$\Rightarrow \quad f(x) \approx \frac{2-x}{2}$$

For $\sqrt{0.95} = \sqrt{1-0.05}$, so x in the above equation would be 0.05

Hence
$$f(0.05) \approx \frac{2-0.05}{2}$$

$$\approx 1 - 0.025$$

$$\Rightarrow \quad f(0.05) \approx 0.975$$

Upper bound on the error

$$|R_n(x) \le \frac{|f^{(n+1)}(z)(x-a)^{n+1}|}{(n+1)1}$$

Here
$$n = 1$$

$$|R_1(x)| \le \left|\frac{f^2(z)(x-0)^2}{2!}\right|$$

We have $x = 0.05$ and $f'(x) = \frac{-1}{2}(1-x)^{-\frac{1}{2}}$

$$\Rightarrow \quad f''(x) = \frac{1}{2} \times \frac{1}{2}(1-x)^{-\frac{3}{2}}$$

$$\Rightarrow \quad f''(x) = \frac{1}{4}(1-x)^{-\frac{3}{2}}$$

So, $$R_1(x) \le \left|\frac{f^2(z)(0.05)^2}{2!}\right|$$

$$\Rightarrow \quad R_1(x) \le \frac{0.05^2}{2} \times \frac{1}{4}\left|(1-z)^{-\frac{3}{2}}\right|$$

Z is between x and C

Z is between 0.05 and 0

To make $\left|(1-z)^{-\frac{3}{2}}\right|$ as large as possible when $Z = 0.05$

Since the function is increasing.

Hence $R_1(0.05) \le 0.000337$

Q.4. Answer any three of the following : **3×5=15**

(a) Find all asymptotes for :

(i) $y = \dfrac{x^3+5}{x^2}$

Ans. $$y = \frac{x^3+5}{x^2}$$

Vertical asymptotes

$$x^2 = 0 \Rightarrow x = 0$$

Since the numerator has higher degree than the numerator, there is no horizontal asymptote.

Oblique asymptote

$$= x^2 \overline{\big) x^3 + 5}\ \ x$$
$$\underline{x^3}$$
$$\underline{5}$$

So, $$y = x + \frac{5}{x^2}$$

The polynomial part $y = x$ is the oblique asymptote.

(ii) $y = xe^{-2x}$

Ans. $$y = xe^{-2x}$$

$$y = \frac{x}{e^{2x}}$$

There are no vertical asymptotes

For horizontal asymptote

$$y = \lim_{x\to\infty} \frac{x}{e^{2x}}$$

$$= \lim_{x\to\infty} \frac{1}{2e^{2x}} \qquad \left[L'\text{Hopital's Rule} \frac{\infty}{\infty}\right]$$

$\Rightarrow \qquad y = 0$ [horizontal asymptote]

(b) Do the following functions defined by y have an inverse? Why or why not? If yes, find $\frac{dx}{dy}$:

(i) $y = -x^6 + 5; x > 0$

Ans. $$y = -x^6 + 5; x > 0$$

Checking for one-one

If $$f(x_1) = f(x_2)$$

$$\Rightarrow \quad -x_1^6 + 5 = -x_1^6 + 5$$

$$\Rightarrow \quad x_1 = x_2 \text{ since } > 0$$

Onto

$$y - 5 = -x_6$$

$$\Rightarrow \quad (5 - y)^{1/6} = x$$

For all $y > 5$, there doesn't exist on $x \in R^*$

So, function is not onto,

Hence not invertible.

(ii) $y = 4x^5 + x^3 + 3x$

Ans. This is both one-one and onto. Hence it is invertible.

$$\frac{dy}{dx} = 20x^4 + 3x^2 + 3$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{1}{20x^4 + 3x^2 3}$$

(c) Find the intervals where $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Ans. $$f'(x) = 12x^3 - 12x^2 - 24x$$

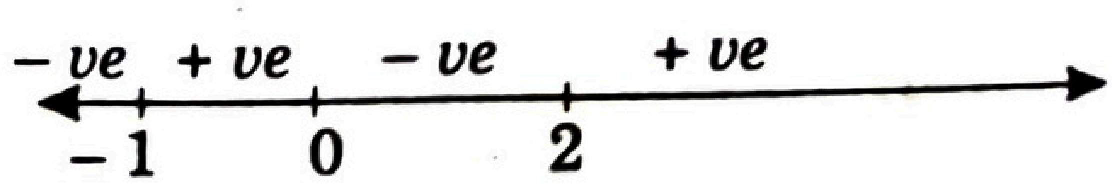
For the function to be increasing, $f'(x) > 0$

$$\Rightarrow \quad 12x^3 - 12x^2 - 24x > 0$$

$$\Rightarrow \quad 12x(x^2 - x - 2) > 0$$

$$\Rightarrow \quad x(x^2 - 2x + x - 2) > 0$$

$$\Rightarrow \quad x(x-2)(x+1) > 0$$



So, the function is increasing on $x \in [-1, 0] \cup [2, \infty]$

and it is decreasing on $x \in [-\infty, -1] \cup [0, 2]$

(d) Calculate the present value of each cash flow using a discount rate of 7% per annum. Which one do you perfer?

Cash flow A : receive ₹ 12 every year, forever, starting today.

Cash flow B : receive ₹ 50 every year for five year, with the first payment being next year. **3×5=15**

Ans. Present value of cash flow A

$$= 12 + \frac{12}{1.07} + \frac{12}{1.07^2} + \ldots$$

$$= 12+\left(\frac{12}{1.07}+\frac{12}{1.07^2}+\dots\right)$$

$$= 12\times\frac{1}{1-\frac{1}{1.07}}$$

$$= 12\times\frac{1.07}{0.07}$$

$$= ₹\ 183.43$$

Present value of cash flow B

$$= \frac{50}{1.07}+\frac{50}{1.07}+\dots+\frac{50}{1.07^5}$$

$$= \frac{50}{1.07}\left[1+\frac{1}{1.07}+\dots\frac{1}{1.07^4}\right]$$

$$= \frac{50}{1.07}\left[\frac{1-\frac{1}{(1.07)^5}}{1-\frac{1}{1.07}}\right]$$

$$= \frac{50}{1.07}\times\frac{1.07}{0.07}\times\frac{1.07^5-1}{1.07^5}$$

$$= \frac{50(1.07^5-1)}{0.07\times 1.07^5}$$

$$= ₹\ 205.01$$

On the basis of present value, Cash flow B would be prefersed to cash flow A.

Q. 5. Answer any three of the following :

(a) Given the function $f(x)\ \frac{1}{x(1-x)}$, what can you say about the existence of extreme point(s) in the interval [2, 3]? Classify the extreme points(s) as local and/or global.

Ans. $$f(x) = \frac{1}{x(1-x)} = \frac{1}{x-x^2}$$

$$\Rightarrow \quad f'(x) = \frac{-1}{(x-x^2)^2}\cdot(1-2x) = \frac{2x-1}{(x-x^2)^2}$$

For extreme points, $f'(x) = 0$

$$\Rightarrow \quad \frac{-(1-2x)}{(x-x^2)^2} = 0$$

$$\Rightarrow \quad x = \frac{1}{2}$$

$$f'\left(\frac{1}{2}\right) = \frac{(x-x^2)^2(2)-(2x-1)(x-x^2)(1-2x)}{(x-x^2)4}$$

$$\Rightarrow \qquad f'\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}-\frac{1}{4}\right)^2\cdot(2)}{\left(\frac{1}{2}-\left(\frac{1}{2}\right)^2\right)^2} > 0$$

So, at $x = \frac{1}{2}$, the function attains the global/local minima.

Also for the interval [2, 3], since this interval does not contain $x = \frac{1}{2}$, we would put the values of $x = 2$ and $x = 2$ and $x = 3$ in the function and check. Although since at $x = \frac{1}{2}$, it has minima, it must be increasing in the interval [2, 3], Hence.

$$f(3) > f(2)$$

$$f(3) = \frac{1}{3\times(-2)} = \frac{1}{6}$$

$$f(2) = \frac{1}{2(-1)} = -\frac{1}{2}$$

(b) Find the interval(s) where the function defined by $y = (x-3)^{2/3}$ is concave/convex. Use this information to find possible points(s) of inflection. Also identify possible cusp(s) in the function. Substantiate your answer with a graph.

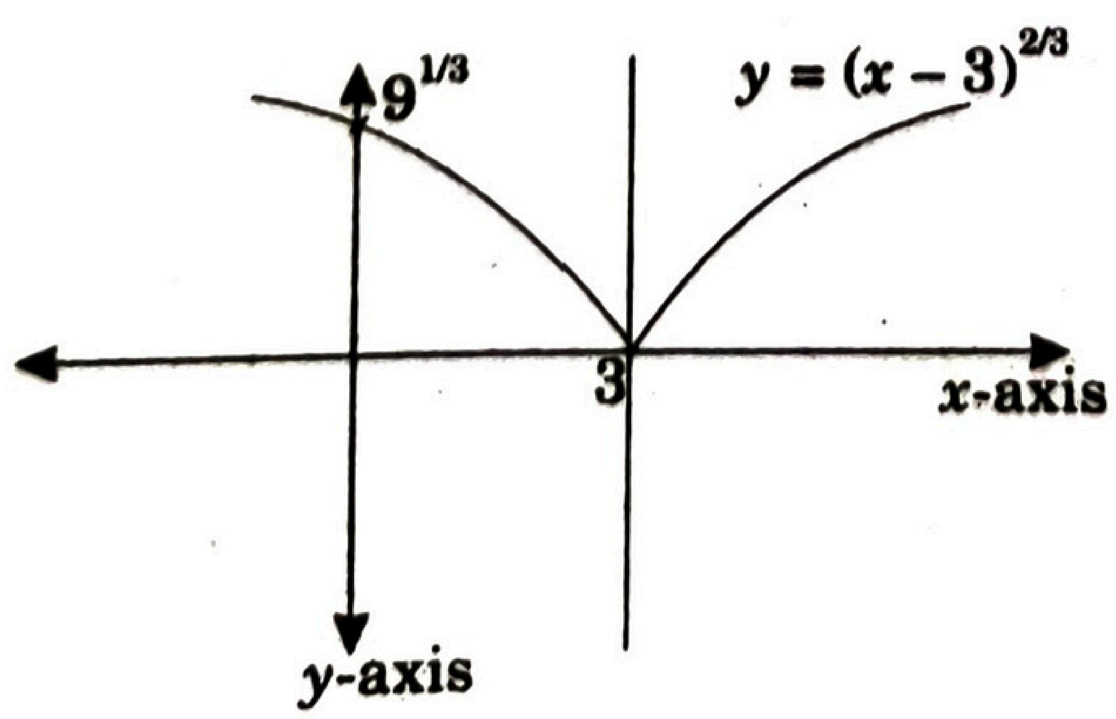
Ans.

$$y = (x-3)^{2/3}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{2}{3}(x-3)^{1/3} = 0 \Rightarrow x = 3$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{2}{9}(x-3)^{-2/3} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=3} = 0$$

Hence at $x = 3$, the function has an inflexion point $\frac{dy}{dx} = \frac{2}{3}(x-3)^{1/2} > 0$ and < 0 for $x < 3$.



(c)A news item is spread by word of mouth to a potential audience of 10,000 people. After t days, $f(t) = \dfrac{1000}{1+50e^{-0.4t}}$ people will have heard the news.

(i) How many people knew about the news at $t = 0$?

Ans.

$$f(t) = \frac{10000}{1+50e^{-0.4t}}$$

$$f(0) = \frac{10,000}{50}$$

$$\Rightarrow \qquad f(0) = 200$$

So, at $t = 0$, 200 will have heard the news.

(ii) When will the news spread at the greatest rate ? (There is no need to check for the sufficient condition here. (Note : ln (5) ≅ 4)).

Ans.

$$f'(t) = \frac{10,000}{(1+50e^{-0.4t})^2}(-50\times 0.4e^{-0.4t})$$

$$\Rightarrow \qquad f'(t) = \frac{2,00,000e^{-0.4t}}{(1+50e^{-0.4t})} > 0 \forall t$$

$$\Rightarrow \quad f''(t) = \frac{[(1+50e^{-0.4})^2(-0.4)e^{-0.4t} \cdot 2(1-50e^{-0.4t})\cdot(50\times(-0.4e^{-0.4t})}{(1+50e^{-0.4t})^4}$$

$$\Rightarrow f''(t) = \frac{2,00,00[-0.4e^{-0.4t}(1+50e^{-.04})^2 + 2\times 0.4\times 50(e^{-0.4t})^2(1+50e^{-0.4t})]}{(1+50\quad ^{0.4})}$$

$$\Rightarrow \quad f''(t) = \frac{2,00,00(0.4)e^{-0.4t}]-(1+50e^{-0.4})+100e^{-0.4t}]}{(1+50e^{-0.4t})^4}$$

$$\Rightarrow \quad f''(t) = \frac{2,00,00(0.4)e^{-0.4t}(1+50e^{-0.4t})[50e^{-0.4}-1]}{(1+50e^{-0.4t})^4}$$

Putting $f''(t) = 0$ for the maximum rate, we get

$$50e^{-0.4t} - 1 = 0$$

$$\Rightarrow \qquad e^{-0.4t} = \frac{1}{50}$$

$$\Rightarrow \qquad e^{0.4t} = 50$$

$$\Rightarrow \qquad 0.4t = \log_e(5)$$

$$\Rightarrow \qquad t = \frac{4}{0.4}$$

$$\Rightarrow \qquad t = 10$$

(iii) Show that $f'(t) = 0.4 f(t)\left[1-\dfrac{f(t)}{10,000}\right]$. Use this formula to calculate $f'(t)$ when $f(t) = 5000$.

Ans.

$$f(t) = \frac{10,000}{1+50e^{-0.4t}}$$

$$\Rightarrow \quad f'(t) = \frac{10,000(50\times(-0.4)e^{-0.4t}}{(1+50e^{-0.4t})^2}$$

$$= \frac{0.4(500000e^{-0.4t})}{(1+50e^{-0.4t})^2}$$

$$= \frac{0.4\times 10,000}{(1+50e^{-0.4t})}\cdot\frac{50e^{-0.4t}}{(1+50e^{-0.4t})}$$

$$= \frac{0.4f(t)}{1}\left[\frac{1+50e^{-0.4t}-1}{1+50e^{-0.4t}}\right]$$

$$= \frac{0.4f(t)}{1}\left[1-\frac{1}{1+50e^{-0.4t}}\right]$$

Since $\frac{f(t)}{10000} = 1+50\,e^{-0.4t}$

We have $f'(t) = 0.4\,f(t)\left[1-\frac{f(t)}{10,000}\right]$

When $f(t) = 5000, f'(t) = 0.4\times 50,000\left[1-\frac{50,000}{10,000}\right]$

$\Rightarrow \quad f(t) = 2000\times(-4)$

$\Rightarrow \quad f'(t) = -8000$

(d) (i) If function $g(x)$ has a minimum at $x = x_0$ show that $f(g)(x))$ also has a minimum at x_0 where $f''(g(x)) > 0$.

Ans. Let $y = f(g(x))$

$$\Rightarrow \quad \frac{dy}{dx} = f'(g(x)).\,g'(x)$$

For maxmia/minima, $\frac{dy}{dx} = 0$, hence

$$f'(g)(x)).\,g'(x) = 0$$

Now since $g'(x_0) = 0$ as $g(x)$ has its minima at $x = x_0$ we have

$$f'(g(x_0)).\,g'(x_0) = 0$$

Hence, the first order condition is satisfied,

Now, **Second Order Condition**

$$\frac{d^2y}{dx^2} = f''(g(x))\,(g'(x))^2 + f'(g(x))\,g''(x)$$

Now since at $x = x_0$ g has its minima, we have $g''(x_0) > 0$

Since $f'(g(x_0)) > 0$, in the above expression, the second term is positive.

Hence all need is

$$|f''(g(x_0))|\,(g'(x_0))^2 < f'(g(x_0))\,.\,g''(x_0) \text{ if } f'' < 0.$$

(If $f'' > 0$, no such condition is required so that the second derivative at x x_0 is positive.

(ii) Find a point on the curve $y = \sqrt{x}$ that is closed to the point (2, 0).

Ans.

$$y = \sqrt{x}$$

Let the closest point on the curve from (2, 0) is (a, b).

Hence the distance is $\sqrt{(a-2)^2 + (b)^2}$

Since $b = \sqrt{a} \Rightarrow b^2 = a$

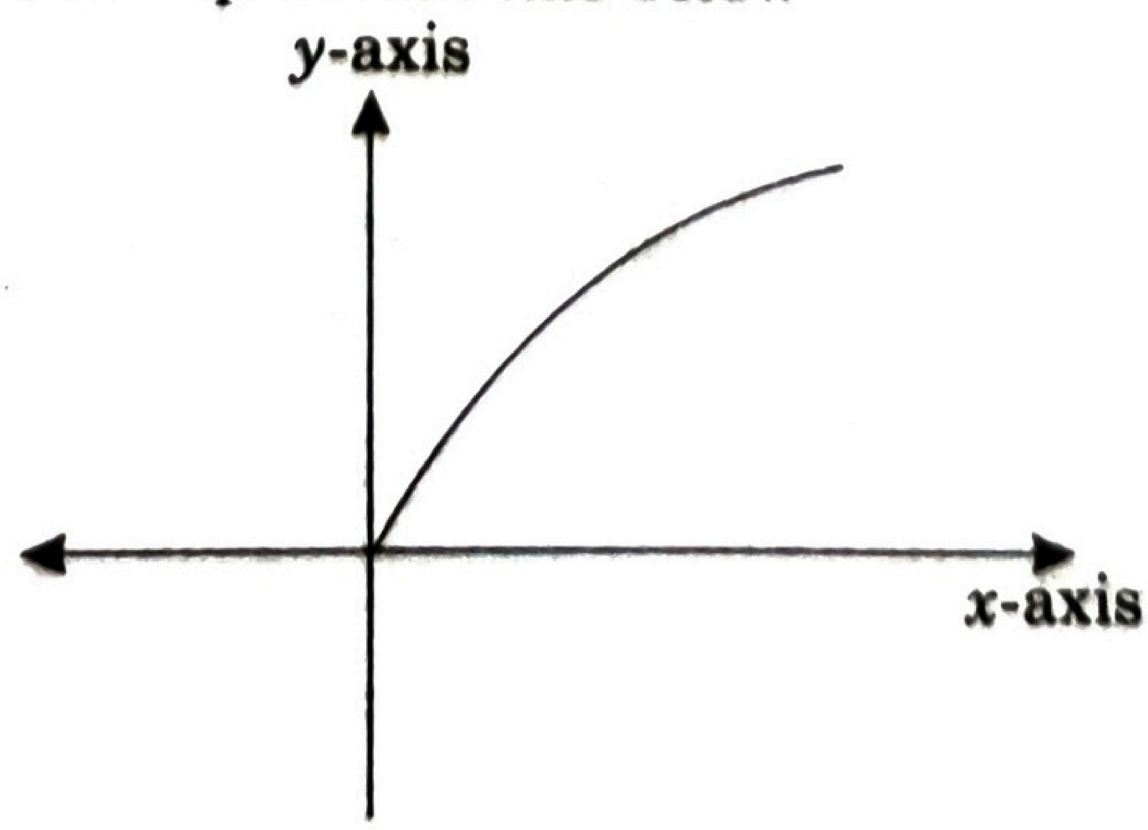
Hence distance

$$
\begin{aligned}
(D) &= \sqrt{(b^2-2)^2 + b^2} \\
\Rightarrow \quad D^2 &= (b^2-2)^2 + b^2 \\
\Rightarrow \quad f(b) &= (b^2-2)^2 + b^2 \\
\Rightarrow \quad f'(b) &= 2(b^2-2)\,.\,2b + 2b \\
&= 2b\,[2b^2 - 4 + 1] \\
&= 2b\,(2b^2 - 3) = 0 \\
\Rightarrow \quad &= 2b = 0 \text{ or } 2b^2 = 3 \\
\Rightarrow \quad b &= 0 \text{ or } b = \pm\sqrt{\frac{3}{2}} \\
f''(b) &= 12b^2 - 6 \\
f''(0) &= -6, f''\left(\pm\sqrt{\frac{3}{2}}\right) = 18-6 \\
\Rightarrow \quad f''\left(\pm\sqrt{\frac{3}{2}}\right) &= 12
\end{aligned}
$$

Hence the minimum distance is from either

$$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right) \text{ or } \left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$$

Since the curve is in I quadrant like below



$$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$$

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